

Formula Sheet for Exam 2 (These formulas will be given.)

The classical wave equation: $\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f(x,t)}{\partial t^2}$

The time-dependent Schrödinger Equation:

$$\boxed{i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi}$$

The standard deviation $\sigma = \sqrt{\langle (\Delta x)^2 \rangle} = \sqrt{\langle (x - \langle x \rangle)^2 \rangle} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

Momentum operator: $\hat{p}_x = \frac{\hbar}{i} \frac{\partial}{\partial x}$

Fourier Transform formulae (Plancherel's Theorem) :

$$f(x) = \frac{1}{\sqrt{2\pi}} \int dk F(k) \exp(ikx)$$

$$F(k) = \frac{1}{\sqrt{2\pi}} \int dx f(x) \exp(-ikx)$$

A useful form of the delta function: $\delta(x) = \frac{1}{2\pi} \int \exp(ikx) dk$

$$J(x,t) \equiv \frac{i\hbar}{2m} \left(\Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x} \right) = \frac{\hbar}{m} \text{Im} \left(\Psi^* \frac{\partial \Psi}{\partial x} \right) = \frac{1}{m} \text{Re} (\Psi^* \hat{p} \Psi)$$

Momentum space wavefunction :

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int dp \Phi(p,t) \exp(ipx/\hbar)$$

$$\Phi(p,t) = \frac{1}{\sqrt{2\pi\hbar}} \int dx \Psi(x,t) \exp(-ipx/\hbar)$$

Position-momentum commutator: $[\hat{x}, \hat{p}_x] = i\hbar$

$$\frac{d\langle Q \rangle}{dt} = \frac{i}{\hbar} \langle [\hat{H}, \hat{Q}] \rangle \quad (\text{assuming } Q \text{ operator is independent of time})$$

Heisenberg Uncertainty Principle: $\sigma_A \sigma_B = \left| \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right| = \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle \right|$

$$[\hat{L}_i, \hat{L}_j] = i\hbar \hat{L}_k$$

Exam 1 Review Topics: Ch 1 and Ch 2 in Griffiths, Homeworks 1 thru 5, and Lecture Notes up thru/including "Dirac Delta function"

- probability density and the wavefunction
- normalization of the wavefunction
- computation of expectation values and standard deviation, given a wavefunction

Classical wave equation, the form of a traveling wave : $f(x,t) = f(x - v t)$, and superposition of solutions due to linearity of the equation.

- Complex variables: polar form vs. cartesian form; complex conjugation; modulus (amplitude) of a complex variable
- Separation of variables.
- Deriving the TISE from the TDSE, starting with separation of variables
- Relation between solutions to the TISE [$\hat{H}\psi_n(x) = E_n \psi_n(x)$] and solutions to the TDSE [$\Psi(x,t) = \sum_n c_n \exp[-iE_n t / \hbar] \psi_n(x)$.] where expansion coefficients c_n gotten from "Fourier's Trick"
- Solutions of the infinite square well.
- Qualitative solutions to the TISE: Sketching the solutions $\psi_n(x)$, given $V(x)$
- Energy eigenvalue equation and properties of the energy eigenfunctions (they form a complete, orthonormal set)
- Free particles: plane-waves states $\Psi(x,t) = A e^{i(kx - \omega t)}$ and wave packets; phase velocity $v_{\text{phase}} = \omega / k$, vs. group velocity $v_{\text{group}} = d\omega / dk$; relation between free particle wavefunction $\Psi(x,t)$ and the Fourier transform $\phi(x)$.
- Procedure for solving the TISE for specific potentials (such a finite square well or scattering from a step): 1) Write general solutions with unknown constants. 2) Apply boundary conditions to determine the constants.
- Tunneling depth
- Reflection and transmission coefficients and relation to the probability current.

Exam 2 Review Topics: Ch 1, 2, and 3 in Griffiths, Homeworks 1 thru 10, and Lecture Notes up thru/including Angular Momentum (upto p.H-9)

- Review Exam 1 Material! Make sure you understand the things you missed on exam 1.
- Hermitean operators and their properties (real eigenvalues, eigenstates that are complete, orthonormal)
- Vectors spaces and Hilbert Space
- The Postulates:

P1: Physical states represented by square-integrable vectors in Hilbert Space

P2: Observables are represented by Hermitean operators. The result of a measurement is always one of the eigenvalues. Operators for momentum p, position x, and any function of x, p

$$P3: \text{Prob}(\text{find } q_n) = |\langle f_n | \Psi \rangle|^2 \quad \text{Prob}(\text{find } q \rightarrow q + dq) = |\langle f_q | \Psi \rangle|^2 dq$$

P4: Wavefunction collapse

P5: TDSE

- Particle in a 3D box. Degeneracy.
- Commutation relations, operator algebra
- If operators commute, they can have simultaneous eigenfunctions.
- Time-dependence of expectation values
- Heisenberg Uncertainty Principle and time-energy uncertainty
- Angular Momentum operator $\hat{\mathbf{L}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}}$
- Operator algebra: $[A+B, C] = [A, C] + [B, C]$; $[AB, C] = A[B, C] + [A, C]B$
- Angular momentum raising and lowering operators and proof of angular momentum eigenvalue equations: $\hat{L}^2 Y = \hbar^2 \ell(\ell+1)Y$, $L_z Y = \hbar m Y$, possible values of ℓ , m .